ESTIMATION OF THE TURBULENCE ENERGY DISSIPATION RATE FROM THE CONICALLY SCANNING PULSED COHERENT DOPPLER LIDAR DATA

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Abstract

In this paper we present the basic relations for the estimation of the turbulence energy dissipation rate from the transverse and longitudinal structure function of the radial wind velocity measured by pulsed coherent Doppler lidar (CDL) at the conical scanning (both one full scan and multiple sectorial scanning). We compare the results of the pulsed CDL profiling the dissipation rate in the the ABL by these methods with the estimation of the dissipation rate obtained from the sonic anemometer data. We present the results of estimating the dissipation rate at the given height from the transverse and from the longitudinal structure function calculated based on the same data of the radial wind velocity measured by CDL during long temporal period and compare these results with each other.

1. Introduction

The use of conical scanning by a sensing beam of the CDL around the vertical axis at the fixed angle of sight allows the information about wind direction and velocity to be obtained. If measurements are conducted by a pulsed CDL, then the vertical profiles of these wind parameters can be reconstructed from the data obtained for one full scan (azimuth angle varies from 0 to 360 degrees). In [1], it is shown for the continuous-wave CDL that not only the wind speed and direction, but also the turbulence energy dissipation rate within the atmospheric boundary layer (ABL) can be estimated from the data measured at the conical scanning. Then in [2], using the approach [1], it is shown that the information about wind turbulence can be retrieved from pulsed CDL data obtained at sectorial conical scanning. In this paper we present the theoretical description of the used approaches for the estimation of the turbulence energy dissipation rate from the transverse and longitudinal structure function of the radial wind velocity measured by pulsed CDL at the conical scanning (both one full scan and multiple sectorial scanning) and the results of the turbulence dissipation rate profiling by these methods.

2. Estimation of the dissipation rate of turbulent energy from scanning CDL data

Consider the case of a 2-mkm pulsed CDL, whose temporal power profile of the sensing radiation is well described by the Gaussian distribution with the pulse duration $2\sigma_P$ determined from the power drop to the e^{-1} level to the left and to the right from the peak. The pulse repetition frequency is denoted as F_P. During measurements by this lidar, the conical scanning with the constant angular rate ω_0 is used. For different distances z_i from the lidar and azimuth angles θ_m, the initial data measured by the lidar are used to calculate Doppler spectra with the use of the rectangular time window with the width T_w for each spectrum and the accumulation of individual estimates of spectra from N_a lidar shots, where $z_i = z_0 + i\Delta R$, $i = 0, 1, 2, ..., I-1$, $z_0 >> \Delta p$, $\Delta p = c\sigma_P / 2$, c is the speed of light, $\Delta R = cT_W / 2$, $\theta_m = \theta_0 + m\Delta\theta$, $m = 0, 1, 2, ..., M-1$, and $\Delta\theta = \omega_0 N_a / F_P$. Then the centroid of the spectral distribution is used to estimate the radial velocity (projection of the wind velocity vector to the axis of the sensing beam) $\hat{V}_r(z_i, \theta_m)$ with allowance made for the Doppler relation. If the estimate is unbiased (the probability of poor estimate caused by the system noise with allowance for the Doppler relation is equal to zero), then $\hat{V}_r(z_i, \theta_m)$ can be represented in the following form with allowance for the wind velocity averaging over the azimuth angle [3, 4]:

$$\hat{V}_r(z_i, \theta_m) = \bar{V}_r(z_i, \theta_m) + V_e(z_i, \theta_m), \quad (1)$$

where

$$\bar{V}_r(z_i, \theta_m) = N_a^{-1} \sum_{k=1}^{N_a} \int_{-\infty}^{+\infty} dz' Q_s(z') V_r(z_i + z', \theta_{m-1} + k\omega_0 / F_P) \quad (2)$$

is the radial velocity averaged over the sensing volume, $Q_s(z')$ is the weight function of averaging along the axis of propagation of the sensing beam, $V_r(z', \theta)$ is the radial velocity at the point $z'\mathbf{S}(\theta)$ of the Cartesian coordinate system $\{z, x, y\}$, $\mathbf{S}(\theta) = \{\sin\varphi, \cos\varphi\cos\theta, \cos\varphi\sin\theta\}$, and $V_e(z_i, \theta_m)$ is the random error of the estimation. This error has the following properties: $<V_e>=0$, $<\bar{V}_r V_e>=<\bar{V}_r><V_e>=0$ and $<V_e(z_i,\theta_m)V_e(z_i,\theta_l)>=\sigma_e^2\delta_{m-l}$, where $\sigma_e^2=<V_e^2>$ is the variance of the random error of estimation of the radial velocity, and δ_{m-l} is the Kronecker delta. Assuming that the pulse repetition frequency F_p is high and the conditions $N_a>>1$ and $z_i>>L_V$ is true, where L_V is the integral scale of correlation of turbulent fluctuations of the wind velocity [5], we pass for the velocity V_r from the polar coordinate system $\{z',\theta\}$ to the rectangular coordinate system on the plane $\{z',y'\}$ (z' is the longitudinal coordinate axis and y' is the transverse axis) and, replacing summation with integration in Eq. (2), we obtain the equation

$$\bar{V}_r(z_i,y_m)=\Delta y^{-1}\int_{-\Delta y/2}^{\Delta y/2}dy'\int_{-\infty}^{+\infty}dz'Q_s(z')\,V_r(z_i+z',y_m+y'), \quad (3)$$

where $y_m=y_0+m\Delta y$ and $\Delta y=z_i\cos\varphi\Delta\theta\cdot\pi/180°$. This approximation is rigorous under the conditions $m\Delta y/(z_i\cos\varphi)<<\pi/2$ and $z_0>>i\Delta R$.

a. Transverse structure function

For the case of horizontally statistically homogeneous and isotropic turbulent flow and small elevation angle φ, from Eqs. (1)-(3) we obtain the equation for the transverse structure function of the radial wind velocity measured by the lidar at the conical scanning, $D_{\hat{V}}(m\Delta y)=<[V_r'(z_i,y_0+m\Delta y)-V_r'(z_i,y_0)]^2>$ ($V_r'=\hat{V}_r-<\hat{V}_r>$, the angular brackets denote the averaging over the ensemble of realizations) in the form

$$D_{\hat{V}}(m\Delta y)=D_{\bar{V}}(m\Delta y)+2(1-\delta_m)\sigma_e^2, \quad (4)$$

where

$$D_{\bar{V}}(m\Delta y)=8\int_0^\infty d\kappa_z\int_0^\infty d\kappa_y S_V(\kappa_z,\kappa_y)H_z(\kappa_z)H_y(\kappa_y)\,[1-\cos(2\pi m\Delta y\kappa_y)] \quad (5)$$

is the transverse structure function of the radial velocity averaged over the sensing volume, $S_V(\kappa_z,\kappa_y)$ is the two-dimensional spatial spectrum of turbulent fluctuations of the wind velocity, $H_z(\kappa_z)$ is the function of the low-pass filter along the axis z' and $H_y(\kappa_y)$ is the function of the low-pass filter along the axis y'.

The analysis of Eqs. (4)-(5) has shown that, when the conditions $\Delta z<2\Delta y$ and $4\Delta y<L_V/2$ are true, the turbulence energy dissipation rate $\hat{\varepsilon}$ can be estimated from experimental estimates of the structure function of the radial velocity $\hat{D}_{\hat{V}}(m\Delta y)$ as

$$\hat{\varepsilon}=\frac{1}{2\Delta y}\left[\frac{\hat{D}_{\hat{V}}(4\Delta y)-\hat{D}_{\hat{V}}(2\Delta y)}{(4/3)(2^{2/3}-1)C_K}\right]^{3/2}. \quad (6)$$

If the condition $4\Delta y<L_V/2$ is not true, then the calculation by Eq. (6) yields the underestimated dissipation rate, and therefore it is necessary to take into account the integral scale of turbulence L_V. By analogy with [2], $\hat{\varepsilon}$ can be estimated by least-square fitting of the experimental structure function $\hat{D}_{\hat{V}}(m\Delta y)$ to the structure function $D_{\hat{V}}(m\Delta y)$ calculated by Eqs. (4), (5). In this case, not only $\hat{\varepsilon}$ is estimated, but also the integral scale of turbulence $\hat{L}_V$ and, according to the von Karman model, the variance of the wind velocity $\hat{\sigma}_V^2=0.636C_K(\hat{\varepsilon}\hat{L}_V)^{2/3}$.

b. Longitudinal structure function

As the measurement range z_i increases, the transverse dimension of the sensing volume $\Delta y=z_i\Delta\theta\cos\phi$ increases as well. However, the condition $z_0>>i\Delta R$ allows us to use the approximation $\Delta y\approx[z_0+\Delta R(I-1)/2]\Delta\theta\cos\varphi$ in Eq. (3), that is, Δy can be considered constant along the propagation path. Then for the longitudinal structure function of the radial velocity $D_{\hat{V}}(i\Delta R)=<[V_r'(z_0+i\Delta R,y_m)-V_r'(z_0,y_m)]^2>$ we obtain from Eqs. (1)-(3)

$$D_{\hat{V}}(i\Delta R)=D_{\bar{V}}(i\Delta R)+2\sigma_e^2[1-K_e(i\Delta R)], \quad (7)$$

where

$$D_{\bar{V}}(i\Delta R)=8\int_0^\infty d\kappa_z\int_0^\infty d\kappa_y S_V(\kappa_z,\kappa_y)H_z(\kappa_z)H_y(\kappa_y)\;[1-\cos(2\pi i\Delta R\kappa_z)]. \quad (8)$$

The analysis of Eqs. (7) and (8) has shown that under the conditions $\Delta y<2\Delta R$, $\Delta z<2\Delta R$, and $4\Delta R<L_V$ the turbulence energy dissipation rate can be estimated from experimental estimates of the longitudinal structure function of the radial velocity $\hat{D}_{\hat{V}}(i\Delta R)$ as [3]

$$\hat{\varepsilon} = \frac{1}{2\Delta R}\left[\frac{\hat{D}_{\hat{V}}(4\Delta R) - \hat{D}_{\hat{V}}(2\Delta R)}{(2^{2/3} - 1)C_K}\right]^{3/2}. \quad (9)$$

At the small integral scale of turbulence (when the condition $4\Delta R < L_V$ is not true), $\hat{\varepsilon}$, $\hat{L}_V$, and $\hat{\sigma}_V^2$ can be estimated with the use of Eq. (8) [4, 5].

3. Experiment

To test the method of estimation of the turbulent energy dissipation rate from lidar measurements of the transverse structure function $\hat{D}_{\hat{V}}(m\Delta y)$, we use data of the experiment conducted in September 2003 in southeastern Colorado within the framework of the Lamar Lower-Level Jet Project [6]. In this experiment, four acoustic anemometers installed on a 120-m meteorological tower (at heights of 54 m, 67 m, 85 m, and 116 m) and 2-mkm pulsed CDL were used. The distance between the mast and the lidar container was 167 m.

During the lidar measurements, different scanning geometries were used. In [7] we used the raw experimental wind data obtained on 15.09.2003 and conducted the comparative analysis of the results of ε estimation from the longitudinal structure function of the radial velocity measured by lidar at scanning by the sensing beam in the vertical plane and from temporal spectra of the wind velocity measured by acoustic anemometers. As a result of comparative analysis of the dissipation rate estimated from lidar data and data of acoustic anemometers measured for the period ~ 16 min, it was found that the relative error of the lidar estimation of ε did not exceed 25% [7].

At the same day during the experiment, to determine wind velocity and direction, one full scan by the sensing beam has been conducted at certain intervals. The results of measurement of the wind velocity and direction, in particular, at the low signal-to-noise ratio, when the filtered sine-wave fitting method was used for the processing of lidar data [8], are reported in [9]. We used these data to retrieve vertical profiles of the dissipation rate with the use of the method of transverse structure function.

One full scan (360° in azimuth) was conducted at the elevation angle $\varphi = 9°$ for one minute. The azimuth resolution was $\Delta\theta = 1.5°$. In this case, at the distance z_i = 1500 m the transverse dimension of the sensing volume Δy was nearly equal to the longitudinal dimension Δz. Therefore, $D_{\bar{V}}(m\Delta y)$ should be calculated by Eq. (5), which takes into account the averaging of the radial velocity over the transverse coordinate y'. The transverse structure function for the height $h_i = h_L + z_i \sin\varphi$ (h_L = 3 m is the height of the lidar) was estimated with the use of averaging over the whole base of the scanning cone and over 5 layers along the sensing beam axis as

$$\hat{D}_{\hat{V}}(m\Delta y) = \frac{1}{5}\sum_{l=1}^{5}\frac{1}{M_s - m}\sum_{m'=1}^{N_s}[\hat{V}_r'(z_i + (l-3)\Delta R, y_0 + (m+m')\Delta y) - \hat{V}_r'(z_i + (l-3)\Delta R, y_0 + m'\Delta y)]^2, \quad (10)$$

where $m = 1, 2, ..., 15$, $M_s = 240$ ($M_s\Delta\theta = 360°$), $\hat{V}_r'(z_i, y_m) = \hat{V}_r(z_i, y_m) - \hat{\mathbf{V}}_i \cdot \mathbf{S}(\theta_m)$, and $\hat{\mathbf{V}}_i = \{\hat{V}_z(h_i), \hat{V}_x(h_i), \hat{V}_y(h_i)\}$ is the estimate of the wind velocity vector obtained as a result of the sine-wave fitting procedure.

Figure 1 shows four vertical profiles of the turbulent energy dissipation rate retrieved from lidar data measured in the nighttime. One can see that the results of measurement by the lidar and by the acoustic anemometers at 02:00 and 03:00 am LT are rather close for all heights of the acoustic anemometers. In general, the depicted results indicate the applicability of the method of transverse structure function of the radial velocity calculated from initial data measured for one full scan to the estimation of the dissipation rate. According to estimates obtained from numerical simulation, the relative error of the lidar estimation of ε varies within 30-40%. An increase in the number of full scans would allow one to obtain data with the greater number of degrees of freedom and, correspondingly, to decrease the error significantly.

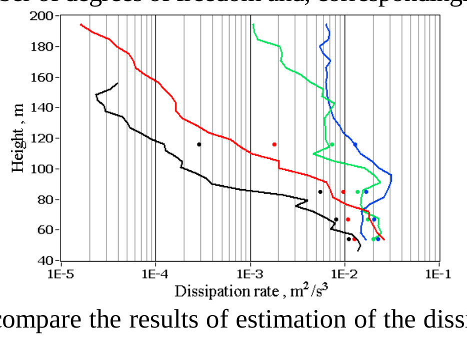

Figure 1. Vertical profiles of the turbulent energy dissipation rate retrieved from data measured by the lidar at the conical scanning by the sensing beam (curves) and four acoustic anemometers on a meteorological tower (circles) on September 15 of 2003 in southeast Colorado (USA). Colors correspond to the measurement time: black – 00:00 am LT, red – 01:00 am LT, green – 02:00 am LT, and blue– 03:00 am LT.

To compare the results of estimation of the dissipation rate by the methods of transverse structure function of the radial velocity calculated from data measured at one full scan and the longitudinal structure function obtained from data of sectorial conical scanning we used the data of the experimental campaign in April of 2011 at a test field of the National Wind Technology Center (NWTC, USA) located about 10 km south of Boulder. The measurements were conducted by the 2-mkm pulsed CDL, whose main parameters can be found in [10].

During the lidar measurements, different geometries of scanning were used in turn. They included the sectorial conical scanning by the sensing beam at different elevation angles and the scanning in the vertical plane at fixed azimuth angles. The full conical scanning was used roughly every half an hour.

To estimate the Doppler spectrum, we used $N_a = 100$ lidar shots. Since the pulse repetition frequency was $F_P = 200$ Hz, the time of measurement of one spectrum (of the radial velocity) was 0.5 s. Lidar estimates of the radial velocity were obtained with a step of $\Delta R = 30$ m along the axis z'. The azimuth resolution $\Delta\theta$ was 0.9º in the case of sectorial scanning, and for full scanning $\Delta\theta$ in different measurements was 2º or 3º.

The sector scanning at elevation angles of 3-3.5º were used for lidar measurements from 19:00 LT on April 14 to 17:30 LT on April 15 of 2011. The duration of individual measurements was, as a rule, from 5 to 12 min.

We used the data measured at the full conical scanning and an elevation angle of 10º every half an hour for 24 hours starting from 18:00 LT of April 14 of 2011 to retrieve the vertical profiles of the wind velocity U and direction θ_V. The same lidar data were also used to retrieve the vertical profiles of the turbulent energy dissipation rate ε with the use of the method of transverse structure function that is described in Section 2 (*a*). The obtained temporal profiles of U, θ_V, and ε at a height $h = 85$ m are shown in Figs. 2 (a), (b), and (c) as blue curves.

The processing procedures described above were used to determine the wind velocity U_a ($h = 85$ m) and direction θ_V, which are shown as red circles in Figs. 2 (a), (b), from the lidar data measured at the sector conical scanning by the sensing beam. The same data were also used to obtain the temporal profile of the turbulent energy dissipation rate ε at a height of 85 m with the use of the method of longitudinal structure function of radial velocity (see Section 2 (*b*)), which is shown as a red curve in Fig. 2 (c). The dissipation rate ε obtained from these data (at sector scanning) by the method of transverse structure function is strongly overestimated in the case of weak turbulence. The results shown as red and blue circles in Figs. 2 (a – c) are in a good agreement for the most of the measurement time.

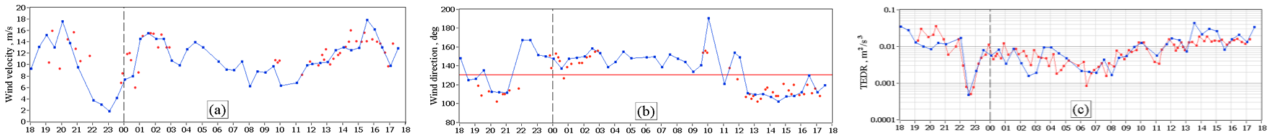

Figure 2. Diurnal profiles of the wind velocity (a), wind direction (b), and turbulent energy dissipation rate (c) at a height of 85 m obtained from the data measured by the lidar at the full conical scanning by the sensing beam (blue curves). Dissipation rate is estimated from the transverse (blue dots) and the longitudinal (red dots) structure functions of the radial velocity calculated from lidar measurements at sector scanning.

5. References

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